Fluid-structure interaction (in OpenFOAM)

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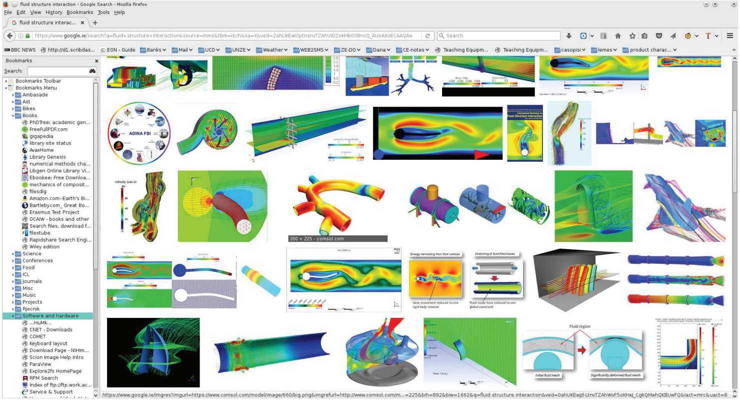
About (computational) FSI

Fluid-structure interaction (FSI) - "a class of problems with mutual dependence between the fluid and structural mechanics parts." [CFSI, 2013]

- Difficult to use analytical methods due to highly nonlinear and time-dependent nature of the problem.
- Very few analytical analytical solutions available (using significant simplifying assumptions).
- Significant developments/advances in computational methods.

Problems and applications

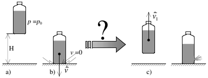
Google/Images: fluid + structure + interaction!!!

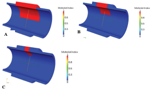


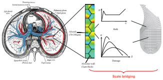
Problems and applications

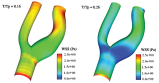
Can be found almost everywhere:

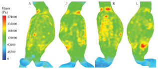
- resistance of fluid-filled plastic bottles (FSI + fracture),
- blood flow through arteries (atherosclerosis, aneurysms, bifurcations, . . .)
- impact resistance of internal organs (lungs),
- pipe fracture analysis,
- aircraft wing flattering,
- deflection of turbine blades,
- parachute dynamics,
- airbag inflation,. . .
- industry, bioengineering, space research, . . .











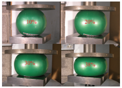


Image sources: [Karac, 2003, Kanyanta, 2011, Kelly, 2009, Quinn, 2011, Parsa, 2012, Alakija, 2005]

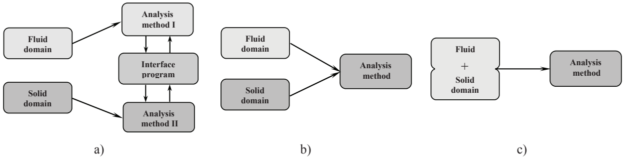
Coupling procedures

Approaches to solve FSI problems:

a) separate analysis (*partitioned*) method / hybrid method (FV-FE, FV-BE, ...)

b) single analysis (*partitioned*) method (FE-FE, FV-FV, FD-FD)

c) single fluid-structure domain (FE, FV, FD)



Breakthrough for (cell-centred) FV-based FSI procedures – (solid) stress analysis using FV methodology [Demirdzic et.al., 1988].

Transport equation

Standard (differential) form:

$$\underbrace{\frac{\partial \phi}{\partial t}}_{\text{temporal derivative}} + \underbrace{\nabla \cdot (\mathbf{v}\phi)}_{\text{convection term}} - \underbrace{\nabla \cdot (\Gamma^{\phi} \nabla \phi)}_{\text{diffusion term}} = \underbrace{Q^{\phi}}_{\text{source term}} \quad (1)$$

Integral form:

$$\int_V \frac{\partial \phi}{\partial t}\, \mathrm{d}V + \oint_S \mathbf{n} \cdot (\mathbf{v}\phi)\, \mathrm{d}S = \oint_S \mathbf{n} \cdot (\Gamma^{\phi} \nabla \phi)\, \mathrm{d}S + \int_V Q^{\phi}\, \mathrm{d}V \quad (2)$$

Parallel unstructured finite-volume method for fluid-structure interaction – a second-order strongly coupled partitioned solution procedure in OpenFOAM [Tukovic et.al., 2018]:

- fluid domain – incompressible fluid model
- solid domain – St. Venant-Kirchhoff material model

Fluid equations (incompressible fluid model)

Mass and linear momentum conservation laws

$$\oint_S \mathbf{n} \cdot \mathbf{v} \, \mathrm{d}S = 0 \tag{3}$$

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \int_V \mathbf{v} \, \mathrm{d}V + \oint_S \mathbf{n} \cdot (\mathbf{v} - \mathbf{v}_s)\mathbf{v} \, \mathrm{d}S \\ &= \oint_S \mathbf{n} \cdot (\nu \nabla \mathbf{v}) \, \mathrm{d}S - \frac{1}{\rho} \int_V \nabla p \, \mathrm{d}V \end{aligned} \tag{4}$$

Geometric (space) conservation law

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \mathrm{d}V - \oint_S \mathbf{n} \cdot \mathbf{v}_s \, \mathrm{d}S = 0 \tag{5}$$

Arbitrary Lagrangian-Eulerian (ALE) formulation.

Solid equations (St. Venant-Kirchhoff model)

The deformation of the solid can be described by the linear momentum conservation law in the **total Lagrangian form** [Tukovic et.al., 2018]:

$$\int_{V_0} \rho_0 \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{u}}{\partial t}\right) \mathrm{d}V = \int_{S_0} \mathbf{n} \cdot \left(\Sigma \cdot \mathbf{F}^{\mathsf{T}}\right) \mathrm{d}S + \int_{V_0} \rho_0 \mathbf{b} \, \mathrm{d}V \tag{6}$$

- $\Sigma = 2\mu \mathbf{E} + \lambda \, \mathrm{tr}\,(\mathbf{E})\, \mathbf{I}$ – the second Piola-Kirchhoff stress tensor
- $\mathbf{E} = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathsf{T}} + \nabla \mathbf{u} \cdot (\nabla \mathbf{u})^{\mathsf{T}}\right]$ – Green-Lagrange strain tensor
- $\mathbf{F} = \mathbf{I} + (\nabla \mathbf{u})^{\mathsf{T}}$ – deformation gradient tensor
- $\sigma = \frac{1}{\det \mathbf{F}} \mathbf{F} \cdot \Sigma \cdot \mathbf{F}^{\mathsf{T}}$ - Cauchy stress tensor

$$\rho_0 \int_{V_0} \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{u}}{\partial t}\right) \mathrm{d}V - \oint_{S_0} \mathbf{n} \cdot (2\mu + \lambda) \nabla \mathbf{u} \, \mathrm{d}S = \oint_{S_0} \mathbf{n} \cdot \mathbf{q} \, \mathrm{d}S + \rho_0 \int_{V_0} \mathbf{b} \, \mathrm{d}V \tag{7}$$

$$\begin{aligned} \mathbf{q} = {} & \mu (\nabla \mathbf{u})^{\mathsf{T}} + \lambda \, \mathrm{tr}\,(\nabla \mathbf{u})\, \mathbf{I} - (\mu + \lambda) \nabla \mathbf{u} \\ & + \mu \, \nabla \mathbf{u} \cdot (\nabla \mathbf{u})^{\mathsf{T}} + \frac{1}{2} \lambda \, \mathrm{tr}\,[\nabla \mathbf{u} \cdot (\nabla \mathbf{u})^{\mathsf{T}}] \mathbf{I} + \Sigma \cdot \nabla \mathbf{u} \end{aligned} \tag{8}$$

Conditions at the FSI interface

The fluid and solid models are coupled by:

- kinematic conditions – velocity and displacements must be continuous across the interface

$$\mathbf{v}_{f,i} = \mathbf{v}_{s,i} \tag{9}$$

$$\mathbf{u}_{f,i} = \mathbf{u}_{s,i} \tag{10}$$

- dynamic conditions – forces at the interface are in equilibrium

$$\mathbf{n}_i \cdot \sigma_{f,i} = \mathbf{n}_i \cdot \sigma_{s,i} \tag{11}$$

$$\sigma_{f,i} = -p\mathbf{I} + \tau = -p\mathbf{I} + \mu \left[\nabla \mathbf{v} + \nabla \mathbf{v}^{\mathrm{T}} \right] \tag{12}$$

Dirichlet-Neumann procedure at the interface:

- fluid solved for a given velocity/displacement (+moving mesh)
- solid solved for a given traction

Managing data transfer

Meshing strategies:

- identical meshing (and identical indexing)
- arbitrary meshing

Interpolation procedures for arbitrary meshing:

- *Face-interpolation procedure* for the interpolation from the boundary cell faces of the fluid side of the interface to the boundary cell faces of the solid side of the interface – performed using the Generalised Grid Interface (GGI) interpolation
- *Vertex-interpolation procedure* for the interpolation of the vertex-displacement field from the solid side of the interface to the vertices of the fluid side of the interface.
    - Polygonal faces at the solid side of the interface are decomposed into triangles using an additional central point (centroid of the polygonal face).
    - Vertices at the fluid side of the interface are projected to the nearest triangle at the solid side of the interface.
    - Displacement values at the projection points are calculated by linear interpolation using known displacements at the triangle vertices.

FSI loop

Iterative procedure (residual calculation for displacements):

$$\{\mathbf{r}\}_i^k = \{\mathbf{u}\}_{s,i}^k - \{\mathbf{u}\}_{f,i}^k \tag{13}$$

FSI schemes:

- Gauss-Seidel iteration schemes

$$\{\mathbf{u}\}_{f,i}^{k+1} = \{\mathbf{u}\}_{f,i}^k + \omega^{k+1}\{\mathbf{r}\}_i^k \tag{14}$$

  - fixed relaxation
  - convergence acceleration with Aitken relaxation
- interface quasi-Newton with approximation for the inverse of the Jacobian from a least-square model (IQN-LS)

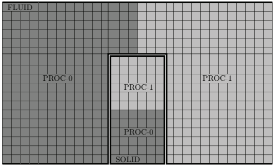
Solution algorithm

Fluid–structure interaction iterative solution procedure

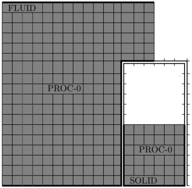
```
1: Switch to the next time step.
2: Predict interface displacement and calculate initial interface residual.
3: Start the FSI strongly coupled iterative procedure.
4: Switch to the next iteration.
5: Calculate the vertex-displacements of the fluid side of the interface.
6: Solve mesh motion equation.
7: Move the fluid mesh.
8: Solve the fluid model.
9: Transfer the face-centre forces from the fluid to the solid side of the interface.
10: Solve the structural model.
11: Transfer the vertex-displacements from the solid to the fluid side of the interface.
12: Calculate interface residual at the fluid side of the interface.
13: if converged then
14:     Go to next time step (line 1)
15: else
16:     Go to next iteration (line 4)
17: end if
```

Parallelisation

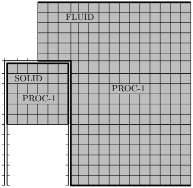
Solver is parallelised using domain decomposition approach.



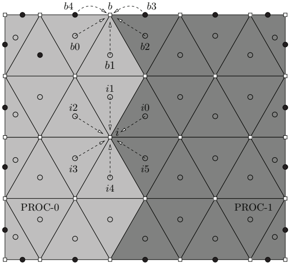
(a) Fluid and solid domains decomposed independently into two sub-domains.



(b) Fluid and solid sub-domains on processor 0.



(c) Fluid and solid sub-domains on processor 1.



OpenFOAM software

The Extend Project (**foam-extend [foam-extend, 2018]** and OpenFOAM® [OpenFOAM, 2018] releases)

- pre-processing: blockMesh utility + third-party with conversion (*e.g.* cfMesh, STAR-CD/PROSTAR, GAMBIT, I-DEAS, CFX)
- solving
  - basic (laplacianFoam, PODSolver, potentialDyMFoam, sixDOFSolver. . .)
  - combustion (dieselFoam, engineFoam, fireFoam, . . .)
  - compressible (sonicFoam, dbnsTurbFoam, sonicLiquidFoam, . . .)
  - coupled (conjugateHeatFoam, MRFPorousFoam, . . .)
  - discreteMethods (molecularDynamics, dsmc)
  - DNS (dnsFoam)
  - electromagnetics (electrostaticFoam, mhdFoam)
  - engine (icoDyMEngineFoam, turbDyMEngineFoam, sonicTurbDyMEngineFoam)
  - equationReaderDemo
  - financial (financialFoam)
  - finiteArea (liquidFilmFoam, surfactantFoam)
  - heatTranfser (boussinesqBuoyantFoam, boussinesqBuoyantPisoFoam, chtMultiRegionFoam, . . .)
  - immersedBoundary (icoDyMIbFoam, porousSimpleIbFoam, . . .)
  - incompressible (icoFoam, pisoFoam, porousSimpleFoam, . . .)
  - lagrangian (coalChemistryFoam, reactingParcelFoam, . . .)
  - multiphase (cavitatingFoam, bubbleFoam, settlingFoam, interFoam, twoLiquidMixingFoam, . . .)
  - multiSolver (multiSolverDemo)
  - solidMechanics [deprecatedSolvers (icoFsiFoam, stressedFoam, . . .), elasticAcpSolidFoam, viscoElasticSolidFoam, elasticOrthoSolidFoam, . . .]
  - surfaceTracking (bubbleInterTrackFoam, interTrackFoam, . . .)
  - viscoElastic (viscoelasticFluidFoam)
- post-processing: third-party (*e.g.* ParaView, EnSight)

Solvers

foam-extend-4.0 for solid/FSI analysis

- solidMechanics – standard solvers for solid and FSI simulations
- extend-bazaar (foam-extend-4.0/extend-bazaar) – self-contained solvers for solid, fluid and fluid-structure interaction (Sorry!!!)
  - solvers – (extend-bazaar/FluidStructureInteraction/)src/solvers
    - fluidFoam
    - solidFoam (+ thermalSolidFoam)
    - fsiFoam (+ ampFsiFoam + weakFsiFoam)
  - (F, S, FSI) models & libraries – (extend-bazaar/FluidStructureInteraction/)src/fluidStructureInteraction
    - fluidSolvers: icoFluid, consistentIcoFluid, . . . + boundary conditions
    - solidSolvers: unsTotalLagrangianSolid, unsIncrTotalLagrangianSolid, . . . + boundary conditions
    - fluidSolidInteraction

Solvers: fluid solver (PISO)

$$\oint_S \mathbf{n} \cdot \mathbf{v} \, \mathrm{d}S = 0 \tag{15}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \mathbf{v} \, \mathrm{d}V + \oint_S \mathbf{n} \cdot (\mathbf{v} - \mathbf{v}_s)\mathbf{v} \, \mathrm{d}S = \oint_S \mathbf{n} \cdot (\nu \nabla \mathbf{v}) \, \mathrm{d}S - \frac{1}{\rho} \int_V \nabla p \, \mathrm{d}V \tag{16}$$

```
fvVectorMatrix UEqn
(
   fvm::ddt(U_)
 + fvm::div(phi_, U_)
 - fvm::laplacian(nu_, U_)
);

solve(UEqn == -gradp_);
volScalarField rAU = 1.0/UEqn.A();
surfaceScalarField rAUf("rAUf", fvc::interpolate(rAU));
```

```
fvScalarMatrix pEqn
(
    fvm::laplacian
    (
       rAUf, p_, "laplacian((1|A(U)),p)"
    )
        == fvc::div(phi_)
);

pEqn.solve();
gradp_ = fvc::grad(p_);
```

Solvers: solid solver

$$\rho_0 \int_{V_0} \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{u}}{\partial t}\right) \mathrm{d}V - \oint_{S_0} \mathbf{n}\cdot(2\mu+\lambda)\nabla\mathbf{u}\,\mathrm{d}S = \oint_{S_0} \mathbf{n}\cdot\mathbf{q}\,\mathrm{d}S + \rho_0 \int_{V_0} \mathbf{g}\,\mathrm{d}V \tag{17}$$

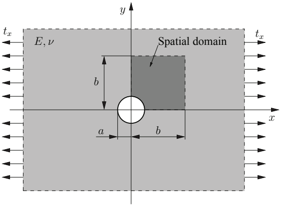
$$\mathbf{q} = \mu(\nabla\mathbf{u})^{\mathrm{T}} + \lambda\,\mathrm{tr}\,(\nabla\mathbf{u})\,\mathbf{I} - (\mu+\lambda)\nabla\mathbf{u} \tag{18}$$

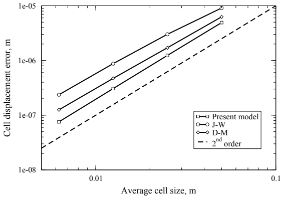
```
fvVectorMatrix DEqn
(
    rho_*fvm::d2dt2(D_)
 == fvm::laplacian(2*muf_ + lambdaf_, D_, "laplacian(DD,D)")
  + fvc::div
    (
        mesh().Sf()
      & (
          - (muf_ + lambdaf_)*gradDf_
          + muf_*gradDf_.T() + lambdaf_*(I*tr(gradDf_))
        )
    )
  + rho_*g
);

solverPerf = DEqn.solve();
```

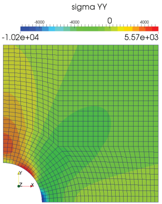
Solvers: FSI solver

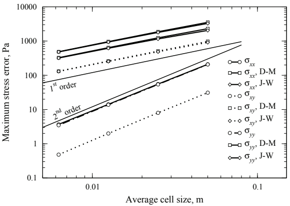
```
for (runTime++; !runTime.end(); runTime++)
{
    fsi.initializeFields();
    fsi.updateInterpolator();
    scalar residualNorm = 0;

    if (fsi.predictor())
    {
       fsi.updateForce();
       fsi.stress().evolve();
       residualNorm = fsi.updateResidual();
    }

    do
    {
       fsi.outerCorr()++;
       fsi.updateDisplacement();
       fsi.moveFluidMesh();
       fsi.flow().evolve();
       fsi.updateForce();
       fsi.stress().evolve();
    }
    while
    (
       (fsi.updateResidual() > fsi.outerCorrTolerance())
       && (fsi.outerCorr() < fsi.nOuterCorr())
    );

fsi.flow().updateFields();
fsi.stress().updateTotalFields();

runTime.write();

Info<< "ExecutionTime = " << runTime.elapsedCpuTime()
    << "  ClockTime = " << runTime.elapsedClockTime()
    << nl << endl;
    }
}
```

Stress analysis: plate with hole



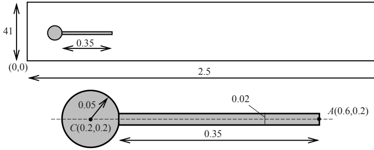


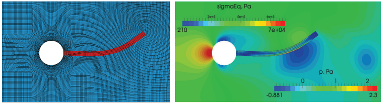




Comparisons with [Demirdzic, Muzaferija, 1995, Jasak, Weller, 2000]

Hron–Turek case





(a) Deformed mesh.

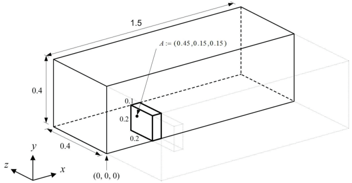
(b) Pressure field in fluid and equivalent stress field in solid.

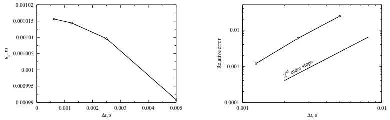
	FSI2	FSI3
ρ_f, kg/m^3	1000	1000
ν_f, m^2/s	0.001	0.001
$\overline{u}$, m/s	1	2
ρ_s, kg/m^3	10000	1000
E_s, Pa	1.4×10^6	5.6×10^6
ν_s	0.4	0.4

	$u_x \times 10^{-3}$ [m]	$u_y \times 10^{-3}$ [m]
FSI2		
Benchmark	$-14.58 \pm 12.44[3.8]$	$1.23 \pm 80.6[2.0]$
Calculated	$-14.26 \pm 12.34[3.9]$	$1.22 \pm 80.2[1.95]$
FSI3		
Benchmark	$-2.69 \pm 2.53[10.9]$	$1.48 \pm 34.38[5.3]$
Calculated	$-2.72 \pm 2.58[11.07]$	$1.67 \pm 33.84[5.53]$
	F_x [N]	F_y [N]
FSI2		
Benchmark	$208.83 \pm 73.75[3.8]$	$0.88 \pm 234.2[2.0]$
Calculated	$211.34 \pm 75.59[3.9]$	$1.23 \pm 238.35[1.95]$
FSI3		
Benchmark	$457.3 \pm 22.66[10.9]$	$2.22 \pm 149.78[5.3]$
Calculated	$459.18 \pm 24.86[11.07]$	$1.59 \pm 155.9[5.53]$

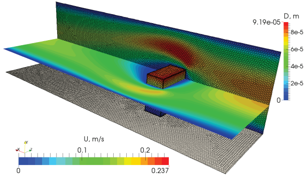
Comparisons with [Turek, Hron, 2006]

Channel flow over elastic plate case



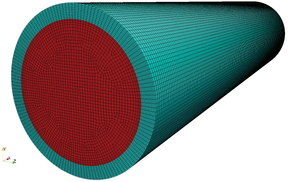


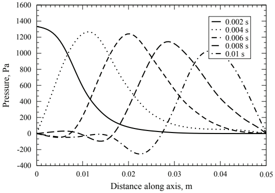
	u_x [m]	u_y [m]	F_x [N]	F_y [N]
Calculated	5.93×10^{-5}	2.40×10^{-5}	1.31	0.1055
Benchmark	5.95×10^{-5}	-	1.33	-

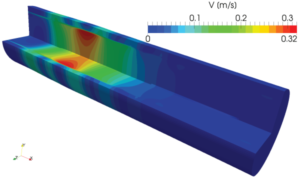


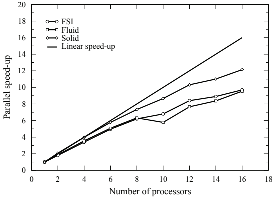
Comparisons with [Richter, 2012]

Elastic tube flow case





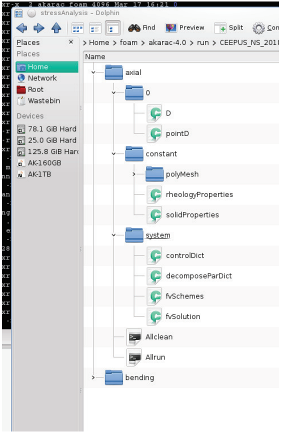




(Solid) Stress analysis

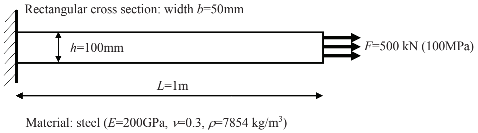
Basic case structure – case dictionaries:

- zero-time dictionary (D, pointD): boundary and initial conditions
- `system` dictionaries
    - `controlDict`: case controls (time, write, . . .)
    - `fvSolution`: solver parameters
    - `fvSchemes`: discretisation schemes (time, gradient, laplacian, . . .)
- `constant` dictionaries
    - `solidProperties`: model properties ⇒ solver
    - `rheologyProperties`: material model
    - `polyMesh/blockMeshDict`: mesh dictionary for `blockMesh`

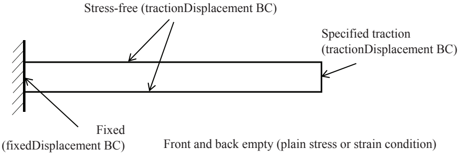


(Solid) Stress analysis

Case study: Axial loading (run/stressAnalysis/axial)



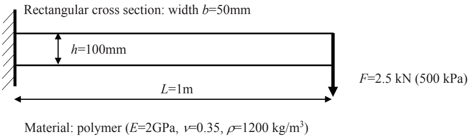
- Boundary and initial conditions (zero-time dictionary: D)



Case 1: Simple steady-state axial loading
Case 2: Dynamic axial loading – stress wave propagation

(Solid) Stress analysis

Case study: (Force) Bending (run/stressAnalysis/bending)

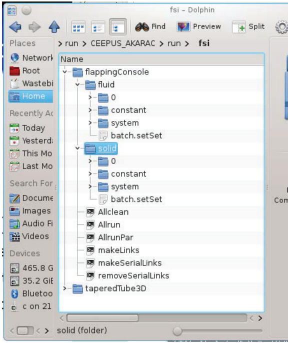


Case 3: Simple steady-state bending
Case 4: Dynamic bending – oscillations/vibrations

Fluid-structure interaction problems

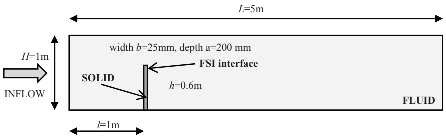
Basic case(s) structure:

- two separate cases/directories
- 'master' domain: fluid (solid part to be linked)
- FSI control parameters and interface set-up (fluid/constant/fsiProperties)

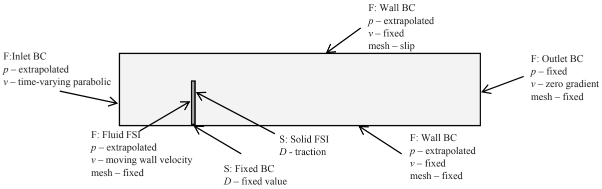


Fluid-structure interaction problems

Case 1: Flapping console

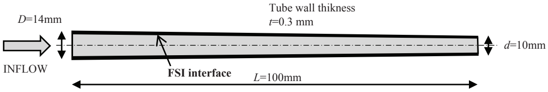


- Boundary conditions



Fluid-structure interaction problems

Case 2: Tapared tube



- Boundary conditions
- Inlet BC

References I

Yuri Bazilevs, Kenji Takizawa, Tayfun E. Tezduyar (2013),
Computational fluid-structure intaraction. *A John Wiley and Sons, Ltd, Publication*.

Karac, A. (2003)
Drop Impact of Polyethylene Containers. *PhD thesis, Imperial College London*.

Kanyanta, V. (2009)
Towards Early Diagnosis of Atherosclerosis: Accurate Prediction of Wall Shear Stress. *PhD thesis, University College Dublin*.

Kelly, S. (2009)
Thrombus Growth and Its Influence on the Stress Distribution in Patient-based Abdominal Aortic Aneurysm Models. *PhD thesis, University College Dublin*.

Quinn, N. (2011)
Towards Early Diagnosis of Atherosclerosis: A Combined Experimental and Numerical Investigation into the Deformation of Mock Arterial Models. *PhD thesis, University College Dublin*.

Parsa, H.K. (2012)
The Development of a Novel Surrogate Lung Material for the Quantitative Prediction of Impact Trauma in Human Lungs. *PhD thesis, University College Dublin*.

Alakija, O.
Modelling Visceral Injuries due to High Rate Impacts to Human Lungs. *Imperial College London*

References II

Tuković, Ž, Karač, A., Cardiff, P., Jasak, H., Ivankoviç, A. (2018),
Parallel unstructured finite-volume method for fluid-structure interaction in OpenFOAM. Accepted for publication in *FAMENA*.

Demirdžić, I., Martinović, D., Ivanković, A (1988),
Numerička simulacija termodeformacionih procesa u zavarenom komadu. *Zavarivanje*, 31:209-219.

Extend-Project (2018)
The foam-extend. *https://sourceforge.net/projects/foam-extend/*.

OpenCFD Ltd, ESI Group (2018)
OpenFOAM®, the open source CFD toolbox. *http://www.openfoam.com*.

Demirdžić, I. and Muzaferija, S. (1995),
Numerical method for coupled fluid flow, heat transfer and stress analysis using unstructured moving meshes with cells of arbitrary topology. *Computer methods in applied mechanics and engineering*, 125:235-255.

Jasak, H. and Weller, H. G (2000),
Application of the finite volume method and unstructured meshes to linear elasticity. *International journal for numerical methods in engineering*, 48:267-287.

Turek, S. and Hron, J.,
Proposal for Numerical Benchmarking of Fluid-Structure Interaction between an Elastic Object and Laminar Incompressible Flow. *in Bungartz, Hans-Joachim and Schfer (eds), Michael: Fluid-Structure Interaction, Vol.53 of Lecture Notes in Computational Science and Engineering,Springer Berlin Heidelberg*, 371-385.

References III

Richter, Th.,
Goal-oriented error estimation for fluid-structure interaction problems. *Computer Methods in Applied Mechanics and Engineering*, 223-224:28-42.